33.16 During design review, an engineer notices a 1-1/4 inch nominal hot water pipe intended to carry $6gpm$ of $130°F$ water takes a sub-optimal route along the perimeter of the building, containing 250 linear feet of pipe and twelve 90 degree elbows. He recommends straightening the run to eliminate the elbows and reduce the total length by 30%. Assuming the pipe is schedule 40 steel with threaded connections, what is the percent reduction in head loss?

A. 22%

B. 33%

C. 44%

D. 55%

The original design, Case 1, requires calculating both the major and minor losses. Case 2 eliminates the elbows such that minor losses can be omitted and reduces the equivalent length. Start by representing the losses for each case accordingly.

$$h_{f,1} = h_{f,1,major} + h_{f,minor} = \frac{fL_1v^2}{2gD} + K\frac{v^2}{2g} = \left(\frac{fL_1}{D} + K\right)\left(\frac{v^2}{2g}\right)$$

$$h_{f,2} = h_{f,2,major} = \frac{fL_2v^2}{2gD} = \left(\frac{fL_2}{D}\right)\left(\frac{v^2}{2g}\right)$$

Represent the percent reduction as the difference over the original. Substitute and algebraically simplify.

$$\frac{h_{f,1} - h_{f,2}}{h_{f,1}} = 1 - \frac{h_{f,2}}{h_{f,1}} = 1 - \frac{\left(\frac{fL_2}{D}\right)\left(\frac{v^2}{2g}\right)}{\left(\frac{fL_1}{D} + K\right)\left(\frac{v^2}{2g}\right)} = 1 - \frac{\left(\frac{fL_2}{D}\right)}{\left(\frac{fL_1}{D} + K\right)}$$

Assume the reduction in losses is not sufficient to change the pipe selection to a smaller diameter; therefore the Reynolds Number, relative roughness, and friction factor will stay roughly the same for the two cases. Use the Steel Pipe Friction Tables, Properties of Water, and Moody Diagram as required.

$$Q = 6gpm$$

$$D = 1.38in\left(\frac{1ft}{12in}\right) = 0.115ft$$

$$v = 1.29\frac{ft}{s}$$

$$\frac{\epsilon}{D} = \frac{.0002ft}{.115ft} \approx 0.002$$

$$Re = \frac{vD}{\nu} = \frac{\left(1.29\frac{ft}{s}\right)(0.115ft)}{0.558 \times 10^{-5}\frac{ft^2}{s}} = 27,000 = 2.7 \times 10^4$$

$$f = f\left(Re, \frac{\epsilon}{D}\right) \approx 0.029$$

For the minor losses applicable to Case 1, look up Threaded Pipe Fittings and find the K value for a 90° standard elbow in a $1\frac{1}{4}in$ diameter pipe. Calculate the total K value accounting for the quantity.

$$K = (12)(1.3) = 15.6$$

Determine the length for Case 2.

$$L_2 = L_1(1 - 30\%) = (250ft)(0.7) = 175ft$$

Substitute and solve for the percent reduction:

$$1 - \frac{\left(\frac{fL_2}{D}\right)}{\left(\frac{fL_1}{D} + K\right)} = 1 - \frac{\left(\frac{(0.029)(175ft)}{(0.115ft)}\right)}{\left(\frac{(0.029)(250ft)}{(0.115ft)} + 15.6\right)} = 0.44 = 44\%$$

Answer C